



CONTRIBUTIONS OF MATHEMATICS GENIUS SRINIVASA RAMANUJAN IN MATHEMATICS

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Abstract

Srinivasa Ramanujan made substantial contributions to the analytical theory of numbers and worked on 'elliptic functions', 'continued fractions', and 'infinite series'. He was a great Mathematician, who became world famous at the tender age of twenty-six. He was born into a family that had a humble background and that had no distinguished professional achievement, yet his mathematical ideas transformed and reshaped 20th century mathematics and continues to inspire modern day mathematicians. Considered to be a mathematical genius, Srinivasa Ramanujan, was regarded at par with the likes of Leonhard Euler and Carl Jacobi. In spite of having almost no formal training in mathematics, Ramanujan's knowledge of mathematics was astonishing. Even though he had no knowledge of the modern developments in the subject, he effortlessly worked out the Riemann series, the elliptic integrals, hypergeometric series, and the functional equations of the zeta function.

Keywords: *Ramanujan, G.H. Hardy, Highly composite numbers, Partition function.*



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Introduction

Srinivasa Ramanujan Iyengar, one of the India's greatest mathematical geniuses, was born on 22nd December 1887 in Erode (Madras Presidency, Tamil Nadu) and grew up in Kumbakonam. When he was about five years old, Ramanujan admitted to the primary school in Kumbakonam although he would attend several different primary schools before entering the Town High School in Kumbakonam in January 1898, where he emerged out to be a scholar student and showed himself as an able all round scholar. Born into a humble family in southern India, he began displaying signs of his brilliance at a young age. He excelled in mathematics as a school student, and mastered a book on advanced trigonometry written by S. L. Loney by the time he was 13. At the age of 16, Ramanujan received a scholarship to study at Government College in Kumbakonam, but lost it when he failed his non-mathematical coursework. He joined another college to pursue independent mathematical research, working as a clerk in the Accountant- General's office at the Madras Port Trust Office to support himself. In 1912–1913, he sent samples of his theorems to three academics

at the University of Cambridge. G. H. Hardy, recognizing the brilliance of his work, invited Ramanujan to visit and work with him at Cambridge. He became a Fellow of the Royal Society and a Fellow of Trinity College, Cambridge, where he was awarded with a degree of B.Sc. (later named Ph.D.) for his work on highly composite number. In 1916, when he was at his best while working with his colleagues Hardy & Littlewood, he met with health problems. He was hospitalized in Cambridge and was diagnosed with tuberculosis and severe vitamin deficiency. After two years struggle, in 1919, he showed some recovery and he decided to return back to India. However, Despite all the tender attention he could get from his wife who nursed him throughout this period, and the best medical attention from the doctors, his untimely end came on 26th April 1920, at Chetput, Madras, when Ramanujan was 32 years old. Even during those months of prolonged illness he kept on working, though in a reclining position, at a furious pace and kept jotting down his results on sheets of paper. Without any formal training in pure mathematics, he made extraordinary contributions to mathematical analysis, number theory, infinite series, and continued fractions. Ramanujan developed his own mathematical research in isolation. As a result, he sometimes rediscovered known theorems in addition to producing new work. Due to his great achievements in the field of Mathematics, his birthday, 22 December, is celebrated as 'State IT Day' in his home state of Tamil Nadu. On the 125th anniversary of his birth, India declared his birthday as 'National Mathematics Day'.

Hardy-Ramanujan Number

Once Hardy visited Putney in a taxi cab having number 1729, where Ramanujan was hospitalized. Hardy was very superstitious due to his such nature when he entered into Ramanujan's room, he quoted that he had just came in a taxi cab having number 1729 which seemed to him an unlucky number but at that time, he prayed that his perception may go wrong as he wanted that his friend would get well soon, but Ramanujan promptly replied that this was a very interesting number as it is the smallest number which can be expressed as the sum of cubes of two numbers in two different ways as given below:

$$1729 = 1^3 + 12^3 = 10^3 + 9^3$$

Later some theorems were established in theory of elliptic curves which involves this fascinating number.

Infinite Series for π :

Srinivasa Ramanujan also discovered some remarkable infinite series of π around 1910. The series

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k!)(1103 + 26390k)}{(k!)^4 396^{4k}}$$

computes a further eight decimal places of π with each term in the series. Later on, a number of efficient algorithms have been developed by number theorists using the infinite series of π given by Ramanujan.

Goldbach's conjecture

Goldbach's conjecture is one of the important illustrations of Ramanujan contribution towards the proof of the conjecture. The statement is every even integer > 2 is the sum of two primes, that is, $6=3+3$. Ramanujan and his associates had shown that every large integer could be written as the sum of at most four (Example: $43=2+5+17+19$).

Theory of Equations

Ramanujan was shown how to solve cubic equations in 1902 and he went on to find his own method to solve the quadratic. He derived the formula to solve biquadratic equations. The following year, he tried to provide the formula for solving quintic but he couldn't as he was not aware of the fact that quintic could not be solved by radicals.

Ramanujan-Hardy Asymptotic formula

Ramanujan's one of the major work was in the partition of numbers. By using partition function $p(n)$, he derived a number of formulae in order to calculate the partition of numbers. In a joint paper with Hardy, Ramanujan gave an asymptotic formulas for $p(n)$. In fact, a careful analysis of the generating function for $p(n)$ leads to the Hardy-Ramanujan asymptotic formula given by

$$p(n) \sim \frac{1}{4n\sqrt{3}} e^{\pi\sqrt{\frac{2n}{3}}}, \quad n \rightarrow \infty$$

In their proof, they discovered a new method called the 'circle method' which made fundamental use of the modular property of the Dedekind η -function. We see from the Hardy-Ramanujan formula that $p(n)$ has exponential growth. It had the remarkable property that it appeared to give the correct value of $p(n)$ and this was later proved by Rademacher using special functions and then Ken Ono gave the algebraic formula to calculate partition function for any natural number n .

Ramanujan's congruences

Ramanujan's congruences are some remarkable congruences for the partition function. He discovered the congruences

$$p(5n + 4) \equiv 0 \pmod{5}$$

$$p(7n + 5) \equiv 0 \pmod{7}$$

$$p(11n + 6) \equiv 0 \pmod{11}, \forall n \in \mathbb{N}.$$

In his 1919 paper, he gave proof for the first two congruences using the following identities using Pochhammer symbol notation. After the death of Ramanujan, in 1920, the proof of all above congruences extracted from his unpublished work.

Highly Composite Numbers

A natural number n is said to be highly composite number if it has more divisors than any smaller natural number. If we denote the number of divisors of n by $d(n)$, then we say $n \in \mathbb{N}$ is called a highly composite if $d(m) < d(n) \forall m < n$ where $m \in \mathbb{N}$. For example, $n = 36$ is highly composite because it has $d(36) = 9$ and smaller natural numbers have less number of divisors. If

$$n = 2^{k_2} 3^{k_3} \dots p^{k_p} \quad (\text{by Fundamental theorem of Arithmetic})$$

is the prime factorization of a highly composite number n then the primes $2, 3, \dots, p$ form a chain of consecutive primes where the sequence of exponents is decreasing; i.e. $k_2 \geq k_3 \geq \dots \geq k_p$ and the final exponent k_p is 1, except for $n = 4$ and $n = 36$.

Some other contributions

Apart from the contributions mentioned above, he worked in some other areas of mathematics such as hypergeometric series, Bernoulli numbers, Fermat's last theorem. He focused mainly on developing the relationship between partial sums and products of hypergeometric series. He independently discovered Bernoulli numbers and using these numbers, he formulated the value of Euler's constant up to 15 decimal places. He nearly verified Fermat's last theorem which states that no three natural number x, y and z satisfy the equation $x^n + y^n = z^n$ for any integer $n > 2$.

Srinivasa Ramanujan's Publications

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